The Number Framework - Strategies

		Addition and Subtraction	Multiplication and Division	Proportions and Ratios
Counting	**Zero:** **Emergent**	**Emergent** The student is unable to count a given set or form a set of up to ten objects.		
Counting	**One:** **One-to-one Counting**	**One-to-one Counting** The student is able to count a set of objects but is unable to form sets of objects to solve simple addition and subtraction problems.	**One-to-one Counting** The student is able to count a set of objects but is unable to form sets of objects to solve simple multiplication and division problems.	**Unequal Sharing** The student is unable to divide a region or set into two or four equal parts.
Counting	**Two:** **Counting from One on Materials**	**Counting – from One** The student solves simple addition and subtraction problems by counting all the objects, e.g. 5 + 4 as 1, 2, 3, 4, 5, 6, 7, 8, 9. The student needs supporting **materials**, like fingers.	**Counting – from One** The student solves multiplication and division problems by counting one to one with the aid of **materials.**	**Equal Sharing** The student is able to divide a region or set into given equal parts using **materials.** With sets this is done by equal sharing of materials. With shapes symmetry (halving) is used.
Counting	**Three:** **Counting from One by Imaging**	**Counting – from One** The student **images** all of the objects and counts them. The student does not see ten as a unit of any kind and solves multi-digit addition and subtraction problems by counting all the objects.	**Counting – from One** The student **images** the objects to solve simple multiplication and division problems, by counting all the objects, e.g. 4×2 as 1, **2**, 3, **4**, 5, **6**, 7, **8**. For problems involving larger numbers the student will still rely on materials.	**Equal Sharing** The student is able to share a region or set into given equal parts by using **materials** or by **imaging** the materials for simple problems, e.g. $\frac{1}{2}$ of 8. With sets this is done by equal sharing of materials or by imaging. With shapes symmetry is used to create halves, quarters, eighths, etc.
Counting	**Four:** **Advanced Counting**	**Counting On** The student uses counting on or counting back to solve simple addition or subtraction tasks, e.g. 8 + 5 by 8, 9, 10, 11, 12, 13 or 52 – 4 as 52, 51, 50, 49, 48. Initially, the student needs supporting **materials** but later **images** the objects and counts them. The student sees 10 as a completed count composed of 10 ones. The student solves addition and subtraction tasks by incrementing in ones (38, 39, 40,…), tens counts (13, 23, 33,…), and/or a combination of tens and ones counts (27, 37, 47, 48, 49, 50, 51).	**Skip-counting** On multiplication tasks, the student uses skip-counting (often in conjunction with one-to-one counting), e.g. 4×5 as 5, 10, 15, 20. The student may track the counts using materials (eg. fingers) or by imaging.	
Part - Whole	**Five:** **Early Additive Part-Whole**	**Early Addition and Subtraction** The student uses a limited range of mental strategies to estimate answers and solve addition or subtraction problems. These strategies involve deriving the answer from known basic facts, e.g. 8 + 7 is 8 + 8 – 1 (doubles) or 5 + 3 + 5 + 2 (fives) or 10 + 5 (making tens). Their strategies with multi-digit numbers involve using tens and hundreds as abstract units that can be partitioned, e.g. 43 + 25 = (40 + 20) + (3 + 5) = 60 + 8 = 68 (standard partitioning) or 39 + 26 = 40 + 25 = 65 (rounding and compensation) or 84 – 8 as 84 – 4 – 4 = 76 (back through ten).	**Multiplication by Repeated Addition** On multiplication tasks, the student uses a combination of known multiplication facts and repeated addition, e.g. 4×6 as (6 + 6) + (6 + 6) = 12 + 12 = 24. The student uses known multiplication and repeated addition facts to anticipate the result of division, e.g. 20 ÷ 4 = 5 because 5 + 5 = 10 and 10 + 10 = 20.	**Fraction of a Number by Addition** The student finds a fraction of a number and solves division problems with remainders mentally using halving, or deriving from known addition facts, e.g. $\frac{1}{3}$ of 12 is 4 because 3 + 3 + 3 = 9, so 4 + 4 + 4 = 12; e.g. 7 pies shared among 4 people (7 ÷ 4) by giving each person 1 pie, and $\frac{1}{2}$ pie, then $\frac{1}{4}$ pie.
Part - Whole	**Six:** **Advanced Additive (Early Multiplicative) Part-Whole**	**Advanced Addition and Subtraction of Whole Numbers** The student can estimate answers and solve addition and subtraction tasks involving whole numbers mentally by choosing appropriately from a broad range of advanced mental strategies, e.g. 63 – 39 = 63 – 40 + 1 = 24 (rounding and compensating) or 39 + 20 + 4 = 63, so 63 – 39 = 24 (reversibility) or 64 – 40 = 24 (equal additions) e.g. 324 – 86 = 300 – 62 = 238 (standard place value partitioning) or 324 – 100 + 14 = 238 (rounding and compensating).	**Derived Multiplication** The student uses a combination of known facts and mental strategies to derive answers to multiplication and division problems, e.g. $4 \times 8 = 2 \times 16 = 32$ (doubling and halving), e.g. 9×6 is $(10 \times 6) - 6 = 54$ (rounding and compensating), e.g. 63 ÷ 7 = 9 because $9 \times 7 = 63$ (reversibility).	**Fraction of a Number by Addition and Multiplication** The student uses repeated halving or known multiplication and division facts to solve problems that involve finding fractions of a set or region, renaming improper fractions, and division with remainders, e.g. $\frac{1}{3}$ of 36, 3×10 = 30, 36 – 30 = 6, 6 ÷ 3 = 2, 10 + 2 = 12 e.g. $\frac{16}{3} = 5\frac{1}{3}$ (using 5 x 3 = 15) e.g. 8 pies shared among 3 people (8 ÷ 3) by giving each person 2 pies and dividing the remaining 2 pies into thirds (answer: 2 + $\frac{1}{3}$ + $\frac{1}{3}$ = $2\frac{2}{3}$). The student uses repeated replication to solve simple problems involving ratios and rates, e.g. 2:3 → 4:6 → 8:12 etc.
Part - Whole	**Seven:** **Advanced Multiplicative (Early Proportional) Part-Whole**	**Addition and Subtraction of Decimals and Integers** The student can choose appropriately from a broad range of mental strategies to estimate answers and solve addition and subtraction problems involving decimals, integers and related fractions. The student can also use multiplication and division to solve addition and subtraction problems with whole numbers, e.g. 3.2 + 1.95 = 3.2 + 2 – 0.05 = 5.2 – 0.05 = 5.15 (compensation); e.g. 6.03 – 5.8 = , as 6.03 – 5 – 0.8 = 1.03 – 0.8 = 0.23 (standard place value partitioning) or as 5.8 + = 6.03 (reversibility); e.g. + 3.98 = 7.04 as 3.98 + = 7.04, = 0.02 + 3.04 = 3.06 (commutativity). e.g. $\frac{3}{4} + \frac{5}{8} = (\frac{3}{4} + \frac{2}{8}) + \frac{3}{8} = 1\frac{3}{8}$ (partitioning fractions) e.g. 81 – 36 = (9 x 9) – (4 x 9) = 5 x 9 (using factors) e.g. 28 + 33 + 27 + 30 + 32 = 5 x 30 (averaging) e.g. (+7) – (-3) = (+7) + (+3) = (+10) (equivalent operations on integers)	**Advanced Multiplication and Division** The student chooses appropriately from a broad range of mental strategies to estimate answers and solve multiplication and division problems. These strategies involve partitioning one or more of the factors in multiplication and applying reversibility to solve division problems, particularly those involving missing factors and remainders. The partitioning may be additive or multiplicative. e.g. $24 \times 6 = (20 \times 6) + (4 \times 6)$ (place value partitioning) or $25 \times 6 - 6$ (rounding and compensating) e.g. 81 ÷ 9 = 9, so 81 ÷ 3 = 3 x 9 (proportional adjustment) e.g. $4 \times 25 = 100$, so 92 ÷ 4 = 25 – 2 = 23 (reversibility and rounding with compensation) e.g. 90 ÷ 5 = 18 so 87 ÷ 5 = 17 r 2 (rounding and divisibility) e.g. 201 ÷ 3 as 100 ÷ 3 = 33 r 1, 200 ÷ 3 = 66 r 2, 201 ÷ 3 = 67 (divisibility rules)	**Fractions, Ratios, and Proportions by Multiplication** The student uses a range of multiplication and division strategies to estimate answers and solve problems with fractions, proportions, and ratios. These strategies involve linking division to fractional answers e.g. 11 ÷ 3 = $\frac{11}{3} = 3\frac{2}{3}$ e.g. 13 ÷ 5 = (10 ÷ 5) + (3 ÷ 5) = $2\frac{3}{5}$ The student can also find simple equivalent fractions, and rename common fractions as decimals and percentages. e.g. $\frac{5}{6}$ of 24 as $\frac{1}{6}$ of 24 = 4, 5 x 4 = 20 or 24 – 4 = 20 e.g. 3:5 as :40, 8 x 5 = 40, 8 x 3 = 24 so = 24 e.g. $\frac{3}{4} = \frac{75}{100}$ = 75% = 0.75
Part - Whole	**Eight:** **Advanced Proportional Part-Whole**	**Addition and Subtraction of Fractions** The student uses a range of mental partitioning strategies to estimate answers and solve problems that involve adding and subtracting fractions, including decimals. The student is able to combine ratios and proportions with different amounts. These strategies include using partitions of fractions and "ones", and finding equivalent fractions. e.g. $2\frac{3}{4} - 1\frac{2}{3} = 1 + (\frac{3}{4} - \frac{2}{3}) = 1 + (\frac{9}{12} - \frac{8}{12}) = 1\frac{1}{12}$ (equivalent fractions) 20 counters in ratio of 2:3 combined with 60 counters in ratio 8:7 gives a combined ratio of 1:1.	**Multiplication and Division of Decimals/Multiplication of Fractions** The student chooses appropriately from a range of mental strategies to estimate answers and solve problems that involve the multiplication of fractions and decimals. The student can also use mental strategies to solve simple division problems with decimals. These strategies involve the partitioning of fractions and relating the parts to one, converting decimals to fractions and visa versa, and recognising the effect of number size on the answer, e.g. $3.6 \times 0.75 = \frac{3}{4} \times 3.6 = 2.7$ (conversion and commutativity) e.g. $\frac{2}{3}$ x $\frac{3}{4}$ = , as $\frac{1}{3}$ x $\frac{1}{4}$ = $\frac{1}{12}$ so $\frac{2}{3}$ x $\frac{1}{4}$ = $\frac{2}{12}$ so $\frac{2}{3}$ x $\frac{3}{4}$ = $\frac{6}{12}$ = $\frac{1}{2}$ e.g. 7.2 ÷ 0.4 as 7.2 ÷ 0.8 = 9 so 7.2 ÷ 0.4 = 18 (doubling & halving with place value)	**Fractions, Ratios, and Proportions by Re-unitising** The student chooses appropriately from a broad range of mental strategies to estimate answers and solve problems involving fractions, proportions, and ratios. These strategies involve using common factors, re-unitising of fractions, decimals and percentages, and finding relationships between and within ratios and simple rates. e.g. 6:9 as :24, $6 \times 1\frac{1}{2}$ = 9, $\times 1\frac{1}{2}$ = 24, = 16 (between unit multiplying); or 9 x $2\frac{2}{3}$ = 24, 6 x $2\frac{2}{3}$ = 16 (within unit multiplying) e.g. 65% of 24: 50% of 24 is 12, 10% of 24 is 2.4 so 5% is 1.2, 12 + 2.4 + 1.2 = 15.6 (partitioning percentages).